Q. P. Code: 25564

(3hours) [Total marks: 80]

N.B. 1) Question No. 1 is compulsory.
2) Answer **any Three** from remaining
3) Figures to the right indicate full marks

1. a) Find Laplace transform of $f(t) = te^{-3t} \sin t$. 5

b) Obtain Complex form of Fourier series of $f(x) = e^x, -1 < x < 1$ in $(-1, 1)$. 5

c) Does there exist an analytic function whose real part is $u = k(1 + \cos\theta)$? Give justification. 5

d) The equations of lines of regression are $3x + 2y = 26$ and $6x + y = 31$. Find i) means of x and y, ii) coefficient of correlation between x and y. 5

2. a) Evaluate $\int_0^\infty e^t \sin 2t \cos 3t \, dt$. 6

b) Find the image of the square bounded by lines $x = 0, x = 2, y = 0, y = 2$ in the z-plane under the transformation $w = (1 + i)z + 2 - i$. 6

c) Obtain Fourier series of $f(x) = |x|$ in $(-\pi, \pi)$. Hence, deduce that – 8

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

3. a) Find the inverse Laplace transform of $F(s) = \frac{s}{(s^2+9)(s^2+4)}$. 6

b) Solve $\frac{\partial^2 u}{\partial x^2} - 100\frac{\partial u}{\partial t} = 0$, with $u(0,t) = 0, u(1,t) = 0, u(x,0) = x(1-x)$

taking $h = 0.1$ for three time steps up to $t = 1.5$ by Bender –Schmidt method. 6

c) Using Residue theorem, evaluate

i) $\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$ ii) $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2}$ 8

[TURN OVER]

4. a) Solve by Crank –Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$,

$u(0,t) = 0,\ u(5,t) = 100,\ u(x,0) = 20$ taking $h = 1$ for one-time step. 6

b) Obtain the Taylor's and Laurent series which represent the function

$f(z) = \frac{z}{(z-1)(z-2)}$ in the regions, i) $|z| < 1$ ii) $1 < |z| < 2$ 6

c) Solve $(D^2 - 3D + 2)y = 4e^{2t}$ with $y(0) = -3, y'(0) = 5$ where $D \equiv \frac{d}{dt}$ 8

5. a) Find an analytic function $f(z) = u + iv$, if 6

$$u = e^{-x}\{(x^2 - y^2)\cos y + 2xy \sin y\}$$

b) Find the Laplace transform of $f(t) = t\sqrt{1 + \sin t}$ 6

c) Obtain half range Fourier cosine series of $f(x) = x, 0 < x < 2$. Using Parseval's identity, deduce that – 8

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$$

6. a) If $f(a) = \oint_C \frac{3z^2+7z+1}{z-a} dz,\ C: x^2 + y^2 = 4$

find the values of $f(3), f'(1-i)$ and $f''(1-i)$ 6

b) Find the coefficient of correlation between height of father and height of son from the following data, 6

Height of father	65	66	67	68	69	71	73
Height of son	67	68	64	68	72	69	70

c) A tightly stretched string with fixed end points $x = 0$ and $x = l$, in the shape defined by $y = kx(l - x)$ where k is a constant is released from this position of rest. Find $y(x,t)$, the vertical displacement if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. 8