Q. P. Code: 38406

Time:3 hours Total marks: 80

N.B. (1) Question No.1 is compulsory.

(2) Answer any three questions from remaining .

(3) Figures to the right indicate full marks.

Q1.

a) Evaluate $\int_0^\infty \frac{\sin 3t + \sin 2t}{te^t} dt$ 05

b) Find the directional derivative of the function $\phi = 4xz^2 + x^2yz$ at (1,-2,-1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. 05

c) Expand $f(x) = \pi x - x^2$ in a half range sine series in the interval (0,π) 05

d) Show that the function $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic. Find the corresponding analytic function f(z). 05

Q2.

a) Prove that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{\sin x}{x} - \cos x \right]$ 06

b) Find Fourier series to represent $f(x) = 4 - x^2$ in the interval (0,2). 06

c) Solve the following differential equation using Laplace transform $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-t}$, given y(0)=4,y'(0)=2. 08

Q3.

a) Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is conservative. Find the scalar potential for $\bar{F}$ and also find the work done by $\bar{F}$ in moving a particle from(1,0,1) to (2,1,3) 06

b) Obtain the complex form of the Fourier series for $f(x) = e^{3x}$ in (0,3) 06

c) Find the Inverse Laplace Transform of 08

i) $\frac{8s + 20}{s^2 - 12s + 32}$ ii) $\tan^{-1}\left(\frac{s+a}{b}\right)$

Q.4 a) Prove that $\int J_3(x)dx + 2\frac{J_1(x)}{x} + J_2(x) = 0$ 06

b) Evaluate $\int_C (x^2 y dx + x^2 dy)$ where C is the boundary described in the anti clockwise direction of the triangle with vertices (0,0),(1,0) and (1,1). 06

c) Find Fourier series expansion of 08

f(x) = 2   -2< x<0
= x   0<x < 2

Q5. a) Show that the map of the real axis of the z plane is a circle under the transformation $w = \frac{2}{z+i}$ .Find the centre and radius of the circle. 06

b) Find the Fourier Integral representation of 06

$f(x) = 1 \quad |x| < 1$

$=0 \quad |x| > 1$ hence evaluate $\int_0^{\infty} \frac{\sin\omega \cos\omega x}{\omega} d\omega$

c) i) Find the Laplace Transform of 04

$(1 + 2t - t^2 + t^3) H(t-4)$

ii) If $\bar{F} = x^2 z\hat{i} - 2y^3 z^3 \hat{j} + xy^2 z^2 \hat{k}$,find div $\bar{F}$ and curl $\bar{F}$ 04

Q6. a) Use Convolution theorem to find $L^{-1}\left(\frac{s^2}{(s^2+4)^2}\right)$ 06

b)Use Gauss Divergence Theorem to evaluate $\iint_S \bar{N}.\bar{F} ds$ where $\bar{F} = 4x\hat{i} + 3y\hat{j} - 2z\hat{k}$ and S is the surface bounded by x=0,y=0,z=0 and 2x+2y+z=4. 06

c) If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u + v = \cos x \cosh y - \sin x \sinh y$ find $f(z)$ in terms of z 08